SINTRANS is an animation illustrating transformations of the sine function

$$Y = A \text{ SIN } B(X+C) + D$$

Each parameter's effect is observed alone and then collectively and the result is a smooth, continuously changing sine wave. The animation can be stopped at any suitable point and the still picture accurately represents the equation with the given parameter values. For example: if a=2, b=1/2, c=π/2 and d=-1 when the animation is stopped, the equation of the sine wave displayed on the screen would be

$$Y = 2 \text{ SIN } 1/2 \ (X+\pi/2) - 1.$$

Thus, a large variety of sine wave transformations can be examined individually for phase shift, amplitude, frequency and vertical shift. The animation also encourages generalizations which may be difficult to visualize otherwise. For example: How do the graphs of the equations Y = A SIN X and Y = SIN BX compare when parameters A and B approach zero?

Function transformations is a major topic in the mathematics courses of the senior grades in high school. To make the transformations more meaningful, and retainable, one could try a variety of techniques. Among them are: work sheets involving paper and pencil constructions; overhead projections of previously drawn functions; and an oscilloscope and sine wave generator. Each technique has some disadvantages. The paper and pencil contructions can be used to produce any transformation, but are slow and tedious and produce only a single picture. The overhead projections can show continuity in transformations, but are limited to the rigid type. (i.e. reflections, rotations and translations). More versatility is found using the oscilloscope but one is restricted to the sine wave function. It is possible to demonstrate changes in phase, frequency and amplitude more or less simultaneously and with continuity, but borrowing an oscilloscope can prove to be quite inconvenient and the demonstration suffers from a lack of representational information on the display such as labelled axes and equation. SINTRANS is an attempt to overcome these deficeincies. It is meant to be artistically pleasing as well as mathematically accurate so that viewers will enjoy what they see. This was felt to be an important consideration since it is so often true in mathematics.

In carrying out the SINTRANS animation we had 2 objectives:

(1) to determine the effectiveness of using a computer program called ANIM8, developed at Carleton University, to produce animations of mathematical concepts.

(2) to provide us with information on the cost of producing such animations so that we could provide funding agencies, if they exist, with realistic cost estimates of future animations.

We hoped that the cost of producing such animations would prove reasonable enough to convince educational administrators of the cost effectiveness of this method of presentation. We did not attempt to speculate on the cost effectiveness of the approach ourselves because we felt that a comparative study of the relative merits of different methods of teaching was far beyond our capabilities.

The animation was carried out using the computer graphics facilities available on the Sigma 9 computer at Carleton University. These facilities consisted of an animation program, called ANIM8, a version of APL with full graphic capabilities and a disk refreshed television display system for picture output. The ANIM8 program was used for most of the animation but APL proved to be extremely useful for a 4 second sequence showing a straight line fold up to become a circle. The sequence would have been far more difficult to program using ANIM8 which was specifically designed for linear transformations on predefined pictures. This combined use of an efficient Fortran program (ANIM8 was written in Fortran) for well defined problems involving linear transformations, and a high level interpretive language (APL) for complex problems involving nonlinear deformation create a powerfull and comprehensive environment for computer animation. In the future we expect to make more extensive use of APL especially for picture definition.

The following paragraph is an excerpt from the ANIM8 User's Manual and aptly describes the general characteristics of the program.

"ANIM8 is an interactive program for animating simple three dimensional pictures and alphanumeric text. The animator communicates with the program using simple commands which can be easily learned by people with no previous programming experience. ANIM8 has considerable computational capability and is well adapted for animating simulations of complex physical systems. In order to take advantage of existing programs a facility is also provided for communicating the results to ANIM8 where they may be used to control the animation. The pictures produced by ANIM8 are line

drawings. THere is no attempt to shade surfaces or to remove hidden lines from the projections of three dimensional objects.

The pictures are displayed on a television monitor and are captured on film, frame by frame, using a specially adapted camera. For complex images each frame may take several seconds to be composed by the program. As the program completes the frame it triggers the camera and the frame is recorded on film. When the film is subsequently played back at a standard projection rate of 18 or 24 frames per second, the illusion of smooth motion is obtained. The process is interactive in the sense that any frame of the animation may be viewed within a few seconds of the request."

The ANIM8 program was well suited to our application as it permits all picture transformations to be specified in concise mathematical terms using a block oriented language.

A portion of the ANIM8 block network used in SINTRANS to illustrate the effect of changing amplitude is shown in FIGURE 1.

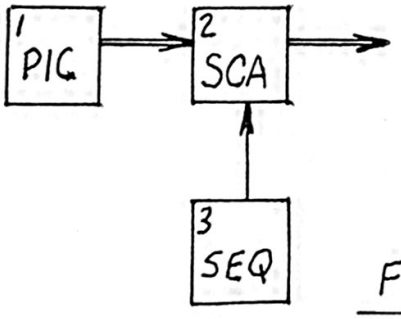

FIGURE 1

The PIC block has a table definition associated with it containing the X,Y,Z coordinates of the points comprising the sine wave. In this case, the table contained 50 points with all the Z coordinates being zero. The SEQ (sequence) block is a tabular function containing values of a time varying vector of 3 parameters at selected instants in time. For values of time not included in the table the vector output will be a linear interpolation, component by component, between the 2 nearest table entries. In this case the vector output wil be used by block 2 (SCA block) as scale parameters for each of the X,Y and Z directions. The single line path between bocks 2 and 3 is an ANIM8 convention used in drawing block diagrams to indicate that a vector rather than a picture is flowing between the 2 blocks. The table definition of block 3 is shown below

TIME	V1	V2	V3
0	1	1	0
2	1	3	0
8	1	-3	0
12	1	1	0

If the network is "RUN" (an ANIM8 command) from 0 to 12 seconds in 1/18 second increments and the output from block 2 is displayed on the screen and filmed, frame by frame, the result will be a sine wave with continuously changing amplitude.

We found the use of a large time shared digital computer with a television display system for graphical output to be an extremely economical animation tool for this type of animation. The availability of adequate core memory and disk storage combined with ease of accessibility make it an attractive alternative to the conventional dedicated graphics processor. THe use of a television display system has a further cost saving advantage in that 2 video signals can easily be mixed and the composite image displayed on one monitor. By storing all the static or background information on one channel, and all the dynamic or animated information on another, only the latter need be redrawn each time and complex backgrounds can be displayed at very little computer cost.

The fact that we could not preview the motion sequences did not prove to be a drawback as we found that the animated sequence was exactly what had been anticipated. We discovered that a good animation is conceived in the mind of the animator by intelligent forethought and we doubt that a trial and error approach with a facility capable of displaying motion sequences at speed could produce better results. In any case, ANIM8 has been implemented on a PDP-15 and a PDP-11 GT40 in addition to the Sigma 9, so the advantages of a dedicated machine are available if required. The version of ANIM8 which runs on the Sigma 9 can output pictures to Computek and Tektronix storage tube terminals in addition to the television disply system. We have not, as yet, attempted to film from these terminals but they are used in the development stages of animations. In the future they could provide ANIM8 users with a remote capability for animation allowing a teacher in a school equipped with such a terminal and an animation camera to produce his/her own animations.

Although ANIM8 was originally intended to be used by people with little or no programming experience it takes considerable ANIM8 experience to use the program effectively. Simple animations like rotating a square in the centre of the screen can be programmed by the novice immediately but there are several skills that require considerable practice to be acquired. In our case we decided that it would take the user far too long to become familiar with the program and that having to keep track of the technical details would probably seriously impair his ability to concentrate on content. For these reasons we felt that the only practical approach was to establish distinct roles of PROGRAMMER and USER. We found the

USER-PROGRAMMER-MACHINE mode of communication to be extremely effective and we found that the dialogue between USER and PROGRAMMER greatly enhanced the quality of the final product. We had been concerned at the outset about the role the programmer should play and we feared that misunderstandings about the actual and perceived limitations of the program might seriously hinder the implementation of the user's ideas. This fear originated not so much with the user as with the programmer who realized from previous experience how difficult it is to explain the true limitations of a program without discouraging the user from making further suggestions. The user must be willing to accept that some things which may appear to be trivial simply cannot be done or require too much effort. On the other hand, the programmer must seriously attempt to implement the user's ideas with as little modification as possible. We adopted these roles and attitudes throughout the project and found them to be very helpful.

The cost of producing SINTRANS can be divided into 2 parts: man costs and machine costs. The former includes preparation of the storyboard, preparation and debugging of the ANIM8 block diagrams and, finally, filming and editing. In SINTRANS there were 20 ANIM8 block networks ranging in size from 20 to 50 blocks. Of these 20 networks only 10 were really unique, the others being slight modifications of the original 10. It took about the same amount of time to debug the networks as it took to create them. The table below shows the time spent and the machine cost. The machine cost is fairly accurate because of the existence of a comprehensive charging algorithm on the Sigma 9 which is respresentative of the cost of the facility to the University.

Cost of Producing SINTRANS

Preparation of storyboard = 10 man/hours

Preparation of ANIM8 networks = 20 man/hours

Filming and editing = 10 man/hours

Machine cost for development = $200.00

Machine cost for filming = $200.00

Although SINTRANS is only 4 minutes long it contains several scenes each of which requires its own ANIM8 block network. An animation with a simpler format would cost considerably less. Not included in the man/hour estimates are several informal conversations that we had about animation in general and about ANIM8 in particular. In other words, the figures represent the time it would take us to do another

similar animation taking into account the experience we have gained on this one.

Sine wave transformations is just one of a number of topics in high school mathematics that lend themselves to an animated presentation. We have several ideas for future animations which we hope to pursue if we can find financial support. Before embarking on the SINTRANS project we approached a few sources for funding but were left with the impression that they would prefer to see something tangible before offering assistance. We went ahead without financial assistance doing all the work in our spare time and using computer resources donated by the University computer center in exchange for some program development. We were fortunate that ANIM8 was still in the final stages of development and that the computer center was willing to give us free computer time for helping debug the program.